Simple Approximations for Optimum Wobble Damping of Rotating Satellites Using a CMG

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A simple control law is analytically derived to approximate the time-optimum wobble damping of rotating satellites with and without despun hubs using a control moment gyroscope (CMG) for systems where the angular momentum of the CMG is very small compared with the angular momentum of the satellite. In this approximate optimum control law, hereafter known as the Omega-Dot Law, the CMG spin axis is maintained along $-\dot{\omega}_T$ for satellites rotating about their maximum moment-of-inertia axis, where ω_T is the projection of the satellite angular velocity on the transverse plane. When the rotor revolves about a minimum moment-of-inertia axis, the spin axis of the CMG is directed along $\dot{\omega}_T$. The approximate energy-sink method is used to develop analytical solutions for the wobble rates. These rates agree within approximately 1.5% with the solutions obtained by numerically integrating the Euler equations of motion. Other results include analytical expressions for the minimum CMG angular momentum required to satisfy the wobble-damping task. A simpler and more practical steering law, known as the 90° H-Lag Law, is shown to be an averaging approach to the Omega-Dot Law and provides almost equal performance for many satellite configurations.

Nomenclature

$E(k)$	= complete elliptic integral of the second kind with modulus k
$F(k)$	= complete elliptic integral of the first kind with modulus k
$G(\alpha)$	= see Eq. (27)
$\mathbf{H}$	= angular momentum of satellite without CMG
$\mathbf{h}$	= angular momentum of CMG
I_x	= principal moment of inertia of satellite rotor about rotor spin axis (x axis)
I_y, I_z	= principal moments of inertia of rotor plus hub about the y and z axes, respectively
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	= unit vectors along the x, y, and z axes, respectively
K	= see Eq. (32)
k	= see Eq. (23)
k'	= see Eq. (25)
p	= linearized value of ω_x, the average spin rate of the satellite
T, T_o	= kinetic energy of satellite, subscript o denotes nominal state (pure spin about x axis with no wobble)
t	= time
W	= work done on satellite
x, y, z	= principal axes of satellite with origin at mass center of entire satellite (rotor plus hub)
α	= see Eq. (2)
β	= phase angle of $\boldsymbol{\omega}_T$
δ	= the angle between $\mathbf{h}$ and $\dot{\boldsymbol{\omega}}_T$ minus $\pi/2$
$\lambda, \lambda_y, \lambda_z$	= see Eq. (2)
Ω	= amplitude of ω_z
$\boldsymbol{\omega}$	= angular velocity of satellite rotor
$\boldsymbol{\omega}_T$	= component of $\boldsymbol{\omega}$ in transverse (y, z) plane
$\omega_x, \omega_y, \omega_z$	= components of $\boldsymbol{\omega}$ on x, y, z axes, respectively

Introduction

ARTIFICIAL gravity produced by rotation may be a requirement for manned satellites of the future. Control-moment gyroscopes (CMG's) are effective devices for eliminating the wobble induced by various disturbances. In a previous analysis,[1] several CMG configurations were investigated for wobble damping two fifty-man Space-Base designs. Wobble induced by docking as well as by cargo transfer and interaction with a mass-balancing system was investigated. Other problems studied encompass the influence of elasticity including a flexible rotor-hub bearing and energy dissipation in the structure. It was determined by simulating the dynamic behavior of these satellites that the most desirable of all CMG configurations considered was that shown in Fig. 1. The wobble vector shown in the figure is the projection of the satellite angular-velocity vector on the transverse plane. The gyroscope angular-momentum vector is maintained in the transverse plane and is driven to lag the wobble vector by ninety degrees in inertial space; therefore, this CMG steering law is called the 90° H-Lag Law. In the present paper an analysis is conducted to determine whether the 90° H-Lag Law can be improved and to obtain a desirable control policy for damping wobble in other satellite configurations.

Analysis

1. Uncontrolled System

The satellite and its coordinate system are shown in Fig. 2. It is assumed that the nominal spin axis is a principal axis for both the hub and the rotor and that hub is symmetrical in the sense that both transverse moments of inertia are equal. As a result, the Euler equations are identical to those for a spinning

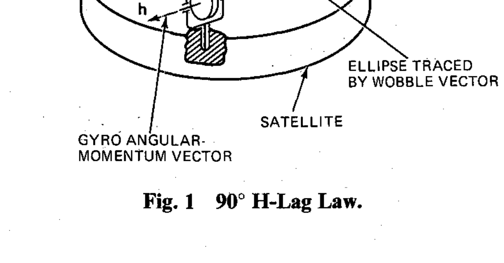

Fig. 1 90° H-Lag Law.

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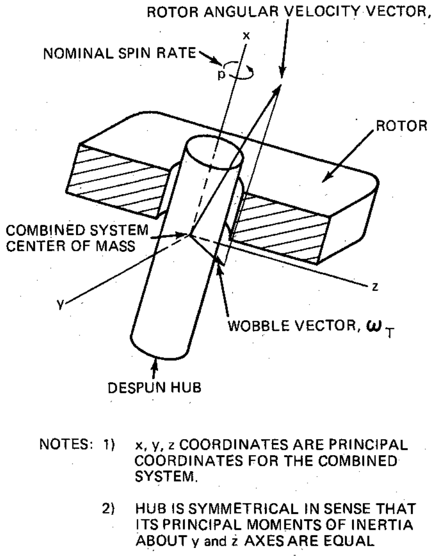

Fig. 2 Satellite with coordinates.

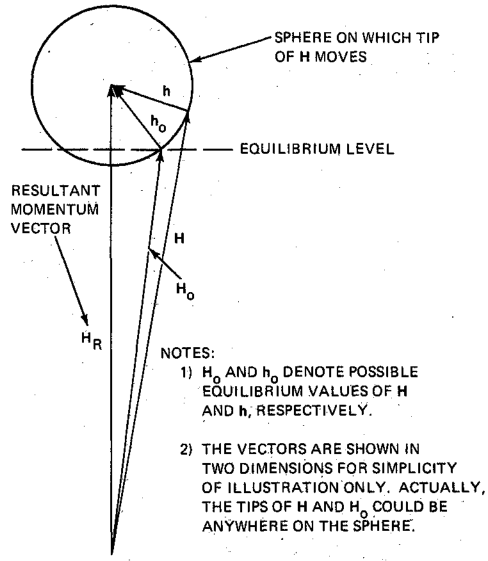

Fig. 3 The angular-momentum vectors in inertial space.

body with no hub provided I_y and I_z include both the hub and rotor mass and I_x includes only the rotor mass.[2] Thus, no further distinction is made between satellites with symmetric hubs and satellites without hubs. The solutions to the linearlized equations for the uncontrolled system are the following:

$$\omega_x = p, \quad \omega_y = \mp \Omega\alpha \sin(\lambda pt + \beta), \quad \omega_z = \Omega \cos(\lambda pt + \beta) \tag{1}$$

where

$$\lambda \equiv (\lambda_y \lambda_z)^{1/2}, \quad \alpha \equiv (\lambda_y/\lambda_z)^{1/2}$$
$$\lambda_y \equiv (I_x - I_z)/I_y, \quad \lambda_z \equiv (I_x - I_y)/I_z \tag{2}$$

and Ω and β are constants of integration. By adjusting β, Ω is taken as positive without loss in generality. The upper sign in the expression for ω_y applies for Max I vehicles ($I_x > I_y, I_x > I_z$), and the lower sign applies for Min I vehicles ($I_x < I_y, I_x < I_z$). This sign convention will be consistently used in the remainder of the paper. The case where I_x is an intermediate axis of inertia is not considered since configurations with this property have unstable motions when the vehicle is uncontrolled.

2. Approximation of the Time-Optimum Steering Law

The angular velocity and angular-momentum vectors of the satellite are

$$\boldsymbol{\omega} = \omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k} \tag{3}$$
$$\mathbf{H} = I_x\omega_x \mathbf{i} + I_y\omega_y \mathbf{j} + I_z\omega_z \mathbf{k} \tag{4}$$

The wobble vector is defined as the projection of $\boldsymbol{\omega}$ on the transverse plane; i.e.,

$$\boldsymbol{\omega}_T = \mathbf{j}\omega_y + \mathbf{k}\omega_z \tag{5}$$

The method which will be used to approximate the optimal control strategy for wobble damping is an extension of the energy-sink method which has often been used to analyze uncontrolled vehicle motion (e.g., see Ref. 3 and 4). From Ref. 2 the kinetic energy of the satellite, with or without control torques, is given by the following expression:

$$2I_x T = H^2 + f(\boldsymbol{\omega}_T) \tag{6}$$

where

$$f(\boldsymbol{\omega}_T) \equiv I_y(I_x - I_y)\,\omega_y^2 + I_z(I_x - I_z)\,\omega_z^2 \tag{7}$$

It is assumed that after the satellite is disturbed from its nominal state of pure spin, no other disturbances occur; thus the resultant angular-momentum vector, $\mathbf{H}_R = \mathbf{H} + \mathbf{h}$ (where $\mathbf{h}$ is the angular momentum of the CMG) is a constant. Consequently, as shown in Fig. 3, the tip of $\mathbf{H}$ moves on a sphere of radius h. $\mathbf{H}_o$ is a possible final equilibrium value of $\mathbf{H}$ which results after the CMG damps all wobble. From Eq. (6), at equilibrium $2I_xT_o = H_o^2$. Any magnitude H_o in the range between $H_R - h$ and $H_R + h$ can be achieved by moving the tail of $\mathbf{h}$ to the corresponding level indicated by the broken line on Fig. 3, and steering the CMG so that $\mathbf{h}$ cones about $\mathbf{H}_R$ at the proper rate such that the energy of the satellite is changed from T to T_o. Thus, unlike the uncontrolled case,[2] in many cases the same satellite can be brought to a state of pure spin by either increasing or decreasing the kinetic energy.

Since the torque which the CMG exerts on the satellite is $-(\dot{\mathbf{h}} + \boldsymbol{\omega} \times \mathbf{h})$, the differential work done by the CMG on the satellite is $dW = -(\dot{\mathbf{h}} + \boldsymbol{\omega} \times \mathbf{h}) \cdot \boldsymbol{\omega}dt = -\dot{\mathbf{h}} \cdot \boldsymbol{\omega}dt$. Integration by parts over one period of the uncontrolled wobble vector $\tau = 2\pi/\lambda p$ yields

$$\Delta W = \int_{t_1}^{t_1+\tau} \mathbf{h} \cdot \dot{\boldsymbol{\omega}} dt - (\mathbf{h} \cdot \boldsymbol{\omega})\Big|_{t_1}^{t_1+\tau} \tag{8}$$

ΔW may be equated to the change in kinetic energy over one cycle as evaluated using Eq. (6). The result is

$$2I_x \int_{t_1}^{t_1+\tau} \mathbf{h} \cdot \boldsymbol{\omega}\, dt - (H^2 + 2I_x \mathbf{h} \cdot \boldsymbol{\omega})\Big|_{t_1}^{t_1+\tau} = f(\boldsymbol{\omega}_T)\Big|_{t_1}^{t_1+\tau} \tag{9}$$

As is common in CMG analyses, it is assumed that $\mathbf{h}$ is along the spin axis of the gyroscope and that its magnitude is constant; i.e., between any time points, $\Delta \mathbf{h}^2 = 0$. In addition, the total momentum of the system is constant; thus over any two time periods $\Delta(\mathbf{H} + \mathbf{h})^2 = 0$. It follows that $\Delta H^2 = -2\Delta(\mathbf{H} \cdot \mathbf{h})$. Substitution of this result and Eqs. (3) and (4) into Eq. (9) yields

$$\Delta U = f(\boldsymbol{\omega}_T)\Big|_{t_1}^{t_1+\tau} \tag{10}$$

where

$$\Delta U \equiv 2I_x \int_{t_1}^{t_1+\tau} \mathbf{h} \cdot \dot{\boldsymbol{\omega}} dt - 2\{[(I_x - I_y)\,\omega_y \mathbf{j} + (I_x - I_z)\,\omega_z \mathbf{k}] \cdot \mathbf{h}\}\Big|_{t_1}^{t_1+\tau} \tag{11}$$

Equation (7) shows that the right side of Eq. (10) is a measure of the change in wobble over one period. ΔU as defined by Eq. (11) is a function of the steering law (i.e., the method of moving $\mathbf{h}$). In order to approximately evaluate these quantities it is assumed that

$$H \gg H_f \gg h \tag{12}$$

where H_f is the maximum acceptable residual angular-momentum component in the transverse plane; i.e., the CMG's function is to damp the wobble angular momentum down to H_f. Since $H \gg h$, the relatively small CMG has very little short-term effect on the motion of the satellite; i.e., the motion of the satellite may be approximated by the uncontrolled motion given by Eq. (1) over one period $\tau = 2\pi/\lambda p$. This simplification also requires $H_f \gg h$, otherwise $\boldsymbol{\omega}_T$ would become so small that the short term perturbations in ω_T due to movement of $\mathbf{h}$ would become significant. Values of H, H_f, and h for the vehicles investigated in Ref. 1 are listed in Table 1. It is seen that the assumed inequalities are justified for these satellites.

Table 1 A comparison of the Angular momenta for the space-base configurations of Ref. 1

Configuration	Angular momenta ft-lb-sec		
	Space-base rotor, H	Maximum acceptable residual component, H_f	CMG, h
50-Man Max-I	241×10^6	1.08×10^4	900
50-Man Min-I	54.7×10^6	10.47×10^4	2500

The uncontrolled values of ω are now used to approximately evaluate Eq. (11). From Eqs. (1) and (5) $\omega = \omega_T + p\mathbf{i}$; thus $\dot{\omega} = \dot{\omega}_T$. $\mathbf{h}$ is assumed to be a function of ω_T, and since ω_T is periodic, $\mathbf{h}$ will also be periodic with the same period (τ). In consequence the last term in Eq. (11) vanishes, and the result is the following approximate relation

$$\Delta U = 2I_x \int_{t_1}^{t_1+\tau} \mathbf{h} \cdot \dot{\omega}_T dt \tag{13}$$

From Eq. (10) and (13) it is seen that the maximum reduction in $f(\omega_T)$ occurs when $\mathbf{h}$ is colinear with $\dot{\omega}_T$. Accordingly, for near-optimum control, the CMG gimbals should be driven to maintain $\mathbf{h}$ in the transverse y, z plane colinear with the wobble acceleration vector $\dot{\omega}_T$. Since for Max-I vehicles Eq. (7) shows that $f(\omega_T) > 0$, $\mathbf{h}$ should be directed oppositely to $\dot{\omega}_T$. Similarly, for Min-I con-configurations, $\mathbf{h}$ should be along $\dot{\omega}_T$. This CMG steering law will be referred to as the Omega-Dot Law.

From Eqs. (1) and (5)

$$\omega_T = \Omega[\mp \alpha \sin(\lambda pt + \beta)\mathbf{j} + \cos(\lambda pt + \beta)\mathbf{k}] \tag{14}$$

Thus

$$\dot{\omega}_T = \lambda p \Omega[\mp \alpha \cos(\lambda pt + \beta)\mathbf{j} - \sin(\lambda pt + \beta)\mathbf{k}] \tag{15}$$

In accordance with the $\dot{\omega}$ law,

$$\mathbf{h} = \mp h\dot{\omega}_T/|\dot{\omega}_T| \tag{16}$$

These results are indicated in Fig. 4 for acute values of $\lambda pt + \beta$. It is seen that the coordinate of $\mathbf{h}$, θ, should be maintained such that

$$\tan\theta = \dot{\omega}_z/\dot{\omega}_y = \pm \alpha^{-1} \tan(\lambda pt + \beta) \tag{17}$$

To compare the Omega-Dot Law with the 90° H-Lag Law of Ref. 1, the angle between h and $\omega_T(\delta + \pi/2$ in Fig. 4) is determined from Eq. (16) which yields $\tan\delta = -(\dot{\omega}_T \cdot \omega_T)/(\mathbf{i} \cdot \dot{\omega}_T \times \omega_T)$. Substituting Eqs. (14) and (15),

$$\tan\delta = \pm \tfrac{1}{2}(\alpha - \alpha^{-1})\sin 2(\lambda pt + \beta); \quad -\pi/2 \leqq \delta \leqq \pi/2 \tag{18}$$

Since δ has an average value of zero, the 90° H-Lag Law can be derived by averaging the gimbal motion used for the $\dot{\omega}$ law. For the symmetric vehicle ($I_y = I_z$, $\alpha = 1$; thus $\delta = 0$) the 90° H-Lag Law of Ref. 1 is indeed equivalent in performance to the $\dot{\omega}$ law. In fact, for the Max-I configuration of Ref. 1, $\alpha = 1.1$, and for the Min-I configuration, $\alpha = 0.97$; therefore the 90° H-Lag Law should yield excellent results. However, from physical considerations, $0 \leqq \alpha \leqq \infty$; thus $\alpha - \alpha^{-1}$ varies between $\pm\infty$, and, accordingly, δ could vary significantly from zero. This can occur for configurations where only one of the transverse principal moments of inertia, I_y or I_z, is near I_x. These inertia properties are characteristic of space-station designs where counterweights are used to increase the distance from the living quarters to the center of rotation during artificial-gravity experiments.

3. Wobble Decay Rates, Average Power Consumption, and Required CMG Angular Momentum

a. *The Omega-Dot Law*

Since $\mathbf{h}$ is to be maintained in the transverse plane, $\mathbf{H}$ and $\mathbf{h}$ are almost perpendicular. In addition $H \gg h$; thus it follows

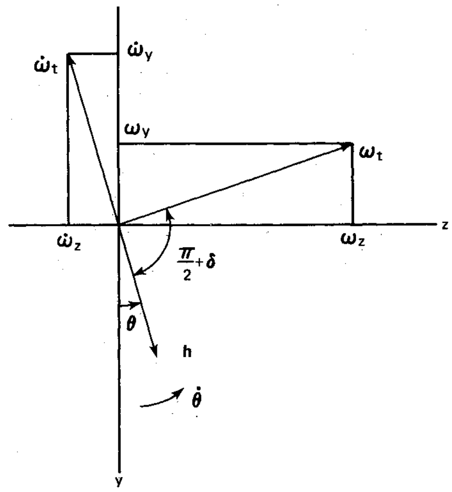

FIGURE 4A. MAX–I CONFIGURATIONS

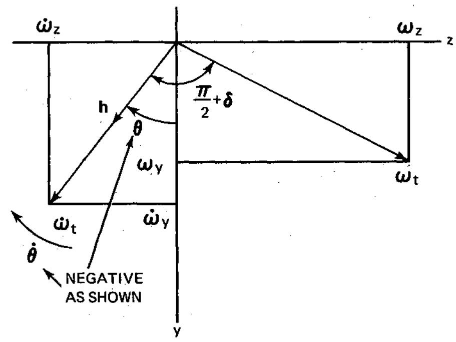

FIGURE 4B. MIN–I CONFIGURATIONS

Fig. 4 Relative positions of wobble vector ω_T and CMG angular momentum vector h in the rotating coordinate system.

from the geometry of Fig. 3 that the magnitude of $\mathbf{H}$ remains nearly constant. Consequently $H \approx H_o$, and subtraction of the equilibrium relation $2I_xT_o = H_o^2$ from Eq. (6) yields the following approximate relation:

$$2I_x(T - T_o) = f(\omega_T) \tag{19}$$

Since $f(\omega_T)$ is positive for Max-I configurations, T_o is a minimum; therefore the energy of the system must be decreased to achieve pure spin. Similarly the energy of Min-I configurations must be increased to achieve pure spin. Thus, when $\mathbf{h}$ is maintained in the transverse plane, the energy-transfer requirements are identical to those for the uncontrolled vehicle.[2]

The energy transferred to the satellite per cycle is evaluated by substitution of Eq. (14–16) into Eq. (8). After some manipulation, the result is

$$\Delta W = \mp h\Omega\alpha \int_0^{2\pi} \left[1 - \left(\frac{\alpha^2 - 1}{\alpha^2}\right)\sin^2\psi\right]^{1/2} d\psi \tag{20}$$

where the dummy variable ψ has replaced $\lambda pt + \beta$. The integral on the right side of Eq. (20) is four times the complete elliptic integral of the second kind; thus

$$\Delta W = \mp 4h\Omega\alpha E(k) \text{ for } 1 \leqq \alpha \leqq \infty \tag{21}$$

where

$$E(k) \equiv \int_0^{\pi/2} \left[1 - k^2 \sin^2\psi\right]^{1/2} d\psi \tag{22}$$

with

$$k \equiv \alpha^{-1}(\alpha^2 - 1)^{1/2} \tag{23}$$

The limits on the range of α in Eq. (21) are imposed since $E(k)$ is only tabulated for $0 \leqq k \leqq 1$. When $0 \leqq \alpha \leqq 1$, Eq. (20) is manipulated into the following expression involving again the tabulated elliptic integral of the second kind:

$$\Delta W = \mp 4h\Omega E(k') \text{ for } 0 \leqq \alpha \leqq 1 \quad (24)$$

where

$$k' \equiv (1 - \alpha^2)^{1/2} \quad (25)$$

Equations (21) and (24) are written compactly to yield the following expression for the work done on the satellite per cycle of the wobble vector:

$$\Delta W = \mp 4h\Omega G(\alpha) \quad (26)$$

where

$$G(\alpha) = \begin{cases} \alpha E(k); & 1 \leqq \alpha \leqq \infty \\ E(k'); & 0 \leqq \alpha \leqq 1 \end{cases} \quad (27)$$

To estimate the average rate of work done on the satellite, Eq. (26) is divided by the wobble period yielding

$$\dot{W} \approx \Delta W/\Delta t = \mp (2/\pi)\, h\lambda p\Omega G(\alpha) \quad (28)$$

An expression for the change in kinetic energy, obtained by substitution of Eqs. (1) and (2) into Eq. (7), and the result into Eq. (19), is

$$2(T - T_o) = (I_z/I_x)(I_x - I_z)\Omega^2 \quad (29)$$

The rate of energy increase is now estimated by permitting Ω, and therefore the amplitudes of the wobble-vector components, to vary slowly with time. Differentiating Eq. (29) yields

$$\dot{T} = (I_z/I_x)(I_x - I_z)\Omega\dot{\Omega} \quad (30)$$

The rate of increase of the satellite's energy given by Eq. (30) is equated to the rate of work done on the satellite given by Eq. (28). A simple differential equation for Ω is obtained which has the following solution:

$$\Omega(t) = \Omega(0) - Kht \quad (31)$$

where

$$K \equiv 2I_x\lambda pG(\alpha)/\pi I_z|I_x - I_z| \quad (32)$$

The wobble vector and the average power consumed by the gimbal torquer motor is estimated by substitution of Eq. (31) into Eq. (14) and (28), respectively, yielding

$$\boldsymbol{\omega}_T = [\Omega(0) - Kht]\,[\mp\alpha \sin(\lambda pt + \beta)\,\mathbf{j} + \cos(\lambda pt + \beta)\,\mathbf{k}] \quad (33)$$

$$\dot{W} = \mp (2/\pi)\, h\lambda pG(\alpha)\,[\Omega(0) - Kht] \quad (34)$$

Since for Max I configurations power is absorbed by the gimbal torquer device ($\dot{W} < 0$), this power could be converted into electricity by using a generator instead of a torquer motor; however, due to hardware limitations, this may not be feasible.

A practical method of specifying the wobble-damping task is to stipulate the reduction in the amplitude of either ω_y or ω_z to be achieved within a specified time t_s. From Eq. (33) the reduction in amplitude of these components are αKht_s *and* Kht_s, respectively, so that the required value of h is

$$h = [\Omega_y(0) - \Omega_y(t_s)]/\alpha Kt_s \text{ or } [\Omega(0) - \Omega(t_s)]/Kt_s \quad (35)$$

where Ω_y is the amplitude of ω_y. Eq. (35) can be used for sizing the CMG.

b. The 90° H-Lag Law

When the 90° H-Lag Law is used, instead of Eq. (16),

$$\mathbf{h} = h(\boldsymbol{\omega}_T \times \mathbf{i})/(|\boldsymbol{\omega}_T|) \quad (36)$$

Substituting Eq. (36) into Eq. (8) and proceeding as previously yields similar results for the wobble vector, the average power consumed by the gimbal torquer, and the required CMG angular momentum. In fact, Eqs. (31–35) are applicable except that $\bar{G}(\alpha)$ replaces $G(\alpha)$ where

$$\bar{G}(\alpha) = \begin{cases} F(k)\,, & 1 \leqq \alpha \leqq \infty \\ \alpha F(k'), & 0 \leqq \alpha \leqq 1 \end{cases} \quad (37)$$

where k and k' are defined by Eqs. (23) and (25), and $F(k)$ is the complete elliptic integral of the first kind; i.e.,

$$\int_0^{\pi/2} \frac{d\psi}{(1 - k^2 \sin^2 \psi)^{1/2}} \quad (38)$$

To compare the effectiveness of the Omega-Dot Law with the 90° H-Lag Law, it is seen from Eq. (32) that the slopes of Ω for the two laws are related by the expression $\bar{K}/K = \bar{G}(\alpha)/G(\alpha)$, where $\bar{K}h$ is the slope for the 90° H-Lag Law. Using the standard expansions for the complete elliptic integrals in terms of their moduli [Eqs. (524) and (525) of Ref. 5]

$$\bar{K}/K = 1 - \tfrac{1}{4}\varepsilon^2 + 0(\varepsilon^3) \quad (39)$$

for both Max-I and Min-I vehicles where $\varepsilon = 1 - \alpha$. It is seen that the term $0(\varepsilon)$ is not present in Eq. (39); therefore when α is near unity, there is no appreciable difference in the damping obtained with the 90° H-Lag Law and that obtained with the Omega-Dot Law. Equation (39) provides the designer with a rapid means for deciding whether it is worthwhile to mechanize the Omega-Dot Law instead of the 90° H-Lag Law.

Table 2 Satellite inertia properties and simulation initial conditions

Satellite configuration	Moment of inertia, 10^6 Slug-ft², I_x	I_y	I_z	Spin rate, rad/sec, p	Simulated wheel angular momentum, h, ft-lb-sec	Simulated initial angular rate $\omega_y(0)$, rad/sec
50-Man Max-I space base	1148.0	853.7	413.7	0.21	1300.0	0.00016
Space-station artificial-g	203.18	202.57	1.7	0.21	130.0	0.01
50-Man Min-I space base	260.6	1234.0	1262.0	0.21	...	...

4. Numerical Verification

Since various approximations have been made in the preceding derivations, the results are verified via a digital simulation using the linearized Euler equations. In addition, wobble-damping performance using the Omega-Dot Law is compared with the 90° H-Lag Law for several different basic rotating-vehicle configurations. The simulation of each law utilized a single-gimbal CMG with its gimbal axis mounted parallel to the vehicle spin axis. In all simulations an initial vehicle rate was imposed about the y axis with zero rate about the z axis. Vehicle descriptive characteristics and initial conditions for all cases studied are shown in Table 2. When wobble damping is no longer required, the gimbal is driven relative to the rotor at a rate equal and opposite to that of the vehicle spin rate. The simulation used a model for the dynamics of the gimbal and torquer-motor combination as shown in Fig. 5.

The first configuration investigated was the 50-Man Max-I Space Base, which is identical to that investigated in Ref. 1. Time-history plots of the wobble rates are shown in Fig. 6*A* for a test case using the Omega-Dot Law. Note that the slopes of the wobble decay-rate amplitudes are straight lines as predicted by Eq. (33). A comparison between the theoretically predicted slope and the measured value is shown in Table 3. The error between predicted and measured slopes is approximately 1.5%. Since this vehicle has an α very close to unity, the theory predicts the difference in damping performance between the Omega-Dot Law and the 90° H-Lag Law to be negligible. Indeed the simulation does exhibit a negligible difference of 0.2%.

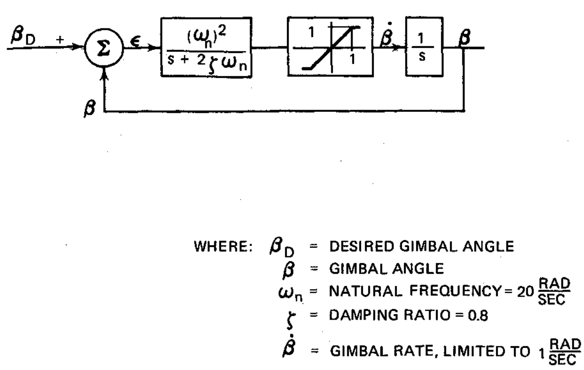

Fig. 5 Math model for CMG torquer motor plus gimbal.

Table 3 Slope of wobble decay–predicted vs measured

Satellite configuration	CMG control law	Slope of y component of wobble decay rate, αKh: Predicted from equations	Measured from simulation
50 Man Max-I	Omega-Dot	0.932×10^{-6}	0.947×10^{-6}
	90° H-Lag	0.930×10^{-6}	0.945×10^{-6}
Space-station	Omega-Dot	21.80×10^{-6}	22.14×10^{-6}
Artificial-g experiment	90° H-Lag	20.46×10^{-6}	20.79×10^{-6}

An additional level of confidence in the theoretical results is obtained by applying the equation for required CMG angular momentum. For the rigid-body 50-Man Max-I and Min-I Space-Base configurations simulated in Ref. 1, the required wheel sizes were estimated to be 1300 ft-1b-sec and 2500 ft-1b-sec, respectively. Substituting the wobble-damping requirements established on p. 10 of Ref. 1 into Eq. (35) yields angular momenta of 1345 ft-1b-sec and 2447 ft-1b-sec, respectively.

Note that when ω_z becomes very small some bumpiness in the curve is indicated in Fig. 6A. This occurs because the perturbations in ω_T induced by movement of the CMG become significant relative to the uncontrolled value of ω_T when ω_T is very small. Thus, in this region we violate the assumption that over short time periods ω_T can be represented by its uncontrolled value. Optimum damping in the region of very small ω_T needs additional study; however, for the satellites investigated to date, damping in this region has not been required.

As noted previously, larger differences in performance between the two control laws are expected to occur for configurations where only one transverse principle inertia, I_y or I_z, is near I_x. These inertia properties are characteristic of a current Space-Station design during its artificial-gravity, experiment (see Table 2). Space-Station time-history plots of the wobble rates are shown in Fig. 6B for a test case using the Omega-Dot Law. Dot Law. The measured slope of the wobble decay and that predicted via the derived formula are tabulated in Table 3. The error between predicted and measured slopes is again only 1.5%. The difference in wobble-damping performance between the two laws is approximately 7% (refer to Table 3) which, although no longer negligible, is not considered appreciable.

Although the Omega-Dot Law is the approximation to the time optimum law, it has disadvantages which are not characteristic of the 90° H-Lag Law. Implementation of the Omega-Dot Law requires angular accelerations to be either sensed or derived, which is more difficult than sensing only angular rates. In addition, the acceleration signal will be oversensitive to small impulsive disturbances and system noise. Furthermore, as the wobble rate is reduced in magnitude, the acceleration vector rapidly changes direction in response to the control torques. In the process of tracking the acceleration vector, the latter phenomenon presents an additional burden on the torquer motor accompanied by an increase in peak power and bearing wear. Due to the above disadvantages and the fact that the 90° H-Lag Law is near optimum for the satellites investigated, the 90° H-Lag Law is considered preferred for these satellites.

References

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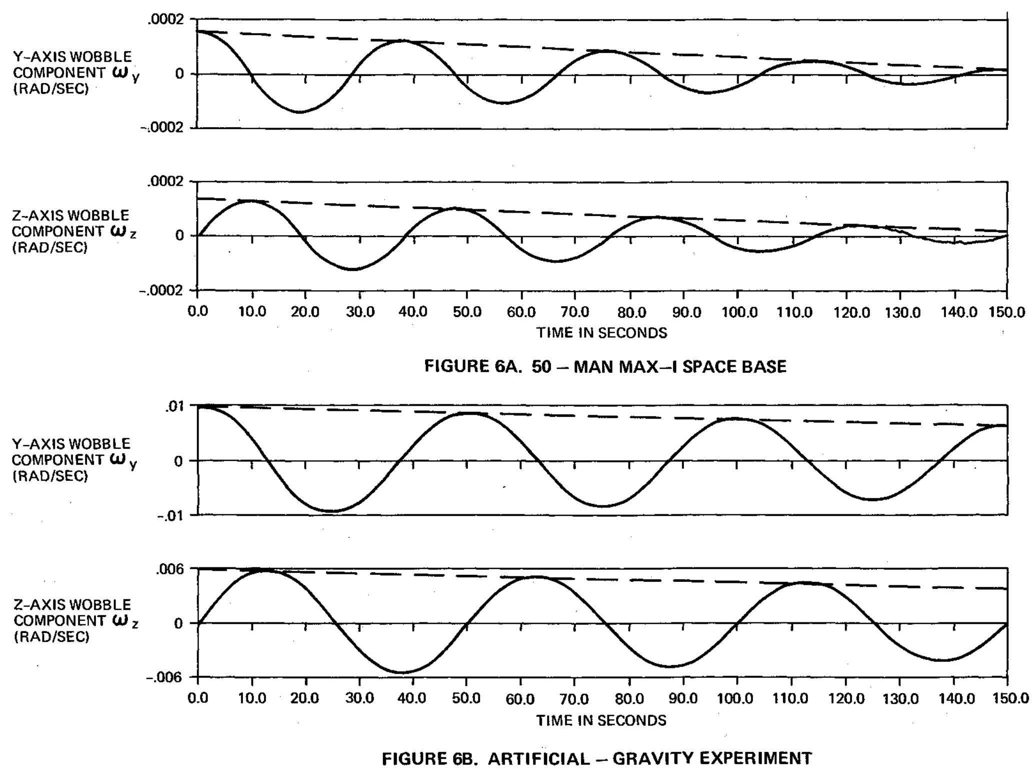

Fig. 6 Time histories of satellite configurations using the Omega-Dot Law.